QBCALC3.TXT
find maximum of quadratic function

$$y = -3x^2 + 24x - 18$$

derivative:

$$\frac{dy}{dx} = -6x + 24$$

0=-6x+24
6x=24
x=4

$$y = 3x^2 + 12x - 4$$

dy/dx=
6x+12
0=6x+12
x=-2
is this minimum or maximum?
derivative of derivative:
second derivative

$$d^2y/dx^2 = 6$$

$$y = -5x^2 - 14x + 3$$

dy/dx=
-10x-14
second derivative=
-10

think of curve as bowl
second deriv. positive: holds water
second deriv. negative: spills water
if first deriv. is zero and:
(a) 2nd deriv. is positive:

local minimum
if first deriv. is zero and:
(b) 2nd deriv. is negative:
local maximum

profit maximization
profit=Z=TR(Q)-TC(Q)

$$\frac{dZ}{dQ} = \frac{dTR}{dQ} - \frac{dTC}{dQ}$$
$$0 = \frac{dTR}{dQ} - \frac{dTC}{dQ}$$

$$0 = \frac{dTR}{dQ} - \frac{dTC}{dQ}$$
$$\frac{dTR}{dQ} = \frac{dTC}{dQ}$$

marginal revenue=marginal cost
profit maximum point:
MR=MC

example: P=85 (constant)
TR=PQ=85Q

$$MR = \frac{dTR}{dQ} =$$

85

$$TC = \frac{1}{15}Q^3 - 2Q^2 + 60Q + 20$$
$$findMC = \frac{dTC}{dQ}$$
$$deriv.of \frac{1}{15}Q^3 :$$
$$\frac{3}{15}Q^2 =$$

$$Q^2/5 = .2Q^2$$
$$deriv.of - 2Q^2 :$$

-4Q
deriv. of 60Q:
60
deriv. of 20:
0

$$MC = .2Q^2 - 4Q + 60$$

at profit maximum: MR=MC

$$85 = .2Q^2 - 4Q + 60$$
$$85 - \mathbf{85} = .2Q^2 - 4Q + 60 - \mathbf{85}$$
$$0 = .2Q^2 - 4Q - 25$$

quadratic equation

$$ax^2 + bx + c = 0$$

quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$
$$.2Q^2 - 4Q - 25 = 0$$

a=
.2
b=
-4
c=
-25

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Q = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(.2)(-25)}}{2(.2)}$$

$$Q = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(.2)(-25)}}{2(.2)}$$

$$Q = \frac{4 \pm \sqrt{16 + 20}}{.4}$$

$$Q = \frac{4 \pm \sqrt{36}}{.4}$$

$$Q = \frac{4 + 6}{.4}$$

$$Q = \frac{10}{.4}$$

Q=25
for profit maximizing problem: use plus sign in quadratic formula

maximize area of rectangle, fixed perimeter
area: a=
=xy
perimeter: p=
=2x+2y
find formula for y:
2y=p-2x
y=.5p-x

a=xy
y=.5p-x
a=x(.5p-x)

$$a = .5px - x^2$$

$$\frac{da}{dx} =$$

=.5p-2x
0=.5p-2x
2x=.5p

x=.25p=p/4
y =
.5p-.25p=(.50-.25)p=.25p
shape:
square

QBCALC5.TXT
two products:
1: extra 2:regular
demand:

$$Q_1 = 29 - 5P_1 + 2P_2$$
$$Q_2 = 13 - 7P_2 + 3P_1$$

cost:
extra: $4, regular$:3
profit:

$$Z = (P_1 - 4)Q_1 + (P_2 - 3)Q_2$$

$$Z = (P_1 - 4)Q_1 + (P_2 - 3)Q_2$$
$$Z = (P_1 - 4)(29 - 5P_1 + 2P_2)+$$
$$(P_2 - 3)(13 - 7P_2 + 3P_1)$$
$$Z = 40P_1 + 26P_2 - 5P_1^2 - 7P_2^2 + 5P_1P_2 - 155$$

two independent variables

$$Z = 40P_1 + 26P_2 - 5P_1^2 - 7P_2^2 + 5P_1P_2 - 155$$
$$pretend that P_2 is constant$$
$$take derivative with respect to P_1$$

called partial derivative

$$Z_{p1}$$

=40

$$-10P_1$$
$$+5P_2$$

$$Z = 40P_1 + 26P_2 - 5P_1^2 - 7P_2^2 + 5P_1P_2 - 155$$

$$partial derivative for P_2 :$$

$$Z_{p2} =$$

26

$$-14P_2$$

$$+5P_1$$

$$Z_{p1} = 40 - 10P_1 + 5P_2$$

$$Z_{p2} = 26 - 14P_2 + 5P_1$$

set both derivatives equal to zero

$$0 = 40 - 10P_1 + 5P_2$$

$$0 = 26 - 14P_2 + 5P_1$$

$$10P_1 - 5P_2 = 40$$

$$-5P_1 + 14P_2 = 26$$

multiply second equation by 2:

$$10P_1 - 5P_2 = 40$$

$$-10P_1 + 28P_2 = 52$$

$$\mathbf{10P_1} - 5P_2 = 40$$

$$-\mathbf{10P_1} + 28P_2 = 52$$

add two equations together:

$$23P_2 = 92$$

$$P_2 = 4$$

$$\begin{gathered}P_2 = 4 \\ solve for P_1: \\ 10P_1 - 5\mathbf{P_2} = 40 \\ 10P_1 - 5(\mathbf{4}) = 40 \\ 10P_1 - 20 = 40 \\ 10P_1 = 60\end{gathered}$$

$$\begin{gathered}P_1 = 6 \\ P_2 = 4\end{gathered}$$