Business 3700 Quantitative Analysis
Practice Assignment – One variable optimization answers

1.

$$P = 41.667 - .611Q$$

$$TR = PQ = (41.667 - .611Q)Q = 41.667Q - .611Q^2$$

$$MR = \frac{dTR}{dQ} = 41.667 - 2 \times .611Q = 41.667 - 1.222Q$$

set the derivative equal to zero:

$$Q = \frac{41.667}{1.222} = 34.097$$

$$P = 41.667 - .611 \times 34.097 = 20.83$$

2.

$$TC = 200 + 100Q + 4.5Q^2$$

$$MC = \frac{dTC}{dQ} = 100 + 9Q$$

$$P = 400 - 3Q$$

$$TR = 400Q - 3Q^2$$

$$MR = \frac{dTR}{dQ} = 400 - 6Q$$

set $MR = MC$:

$$100 + 9Q = 400 - 6Q$$

$$15Q = 300$$

$$Q = 20 \qquad P = 340$$

3.

$$TC = .8Q^3 - 10Q^2 + 48Q + 64$$

$$MC = \frac{dTC}{dQ} = 2.4Q^2 - 20Q + 48$$

set marginal cost equal to price:

$$2.4Q^2 - 20Q + 48 = 60$$

$$2.4Q^2 - 20Q - 12 = 0$$

$$Q = \frac{20 \pm \sqrt{(-20)^2 - 4 \times 2.4 \times (-12)}}{2 \times 2.4} = \frac{20 + \sqrt{515.2}}{4.8} = 8.895$$

4.

$$T = gx + ky + kx + ky$$

Solve for y:

$$2ky = T - gx - kx = T - (g + k)x$$

$$y = \frac{T}{2k} - \frac{g+k}{2k}x$$

$$A = xy = \frac{Tx}{2k} - \frac{g+k}{2k}x^2$$

$$\frac{dA}{dx} = \frac{T}{2k} - \frac{g+k}{k}x$$

set the derivative equal to zero:

$$\frac{T}{2k} - \frac{g+k}{k}x = 0$$

$$\frac{T}{2k} = \frac{g+k}{k}x$$

$$x = \frac{T}{2(g+k)}$$

$$y = \frac{T}{2k} - \frac{g+k}{2k}\left(\frac{T}{2(g+k)}\right) = \frac{T}{2k} - \frac{T}{4k} = \frac{T}{4k}$$

$$\text{if} g = k : \quad x = \frac{T}{4k}$$

$$\text{if} g = 2k : \quad x = \frac{T}{6k}$$

5.

$$S = 2x^2 + 3xy$$

$$y = \frac{S - 2x^2}{3x} = \frac{s}{3}x^{-1} - \frac{2}{3}x$$

$$V = x^2y = x^2\left(\frac{s}{3}x^{-1} - \frac{2}{3}x\right) = \frac{s}{3}x - \frac{2}{3}x^3$$

$$\frac{dV}{dx} = \frac{s}{3} - 2x^2$$

set the derivative equal to zero:

$$0 = \frac{s}{3} - 2x^2$$

$$\frac{s}{3} = 2x^2$$

$$\frac{s}{6} = x^2$$

$$x = \sqrt{\frac{s}{6}}$$

$$y = \frac{s - 2x^2}{3x} = \frac{s - 2 \times \left(\frac{s}{6}\right)}{3\sqrt{\frac{s}{6}}}$$

6.

$$y = 2x^3 - 93x^2 + 1008x + 204$$

$$\frac{dy}{dx} = y' = 6x^2 - 186x + 1008$$

$$\frac{d^2y}{dx^2} = y'' = 12x - 186$$

set the derivative equal to zero:

$$0 = 6x^2 - 186x + 1008$$

Use the quadratic formula:

$$x = \frac{-(-186) \pm \sqrt{(-186)^2 - 4 \times 6 \times 1008}}{2 \times 6}$$

$$x = \frac{186 \pm \sqrt{186^2 - 4 \times 6 \times 1008}}{2 \times 6}$$

$$x = \frac{186 \pm \sqrt{34,596 - 24,192}}{12}$$

$$x = \frac{186 \pm \sqrt{10,404}}{12}$$

$$x = \frac{186 \pm 102}{12}$$

There are two values of x: $x = 24$ and $x = 7$.
When $x = 24$, the second derivative $d^2y/dx^2 = 12 \times 24 - 186 = 102$. Since this is positive, the curve is concave up at this point, so $x = 24$ represents a local minimum.
When $x = 7$, the second derivative $d^2y/dx^2 = 12 \times 7 - 186 = -102$. Since this is negative, the curve is concave down at this point, so $x = 7$ represents a local maximum.

7.
$h = -\frac{1}{2}gt^2 + v_0 t$
$(a)\ \frac{dh}{dt} = -gt + v_0 = -9.8t + 12$
(b) Set the derivative equal to zero:

$$0 = -gt + v_0$$

Solve for t:

$$gt = v_0$$

$$t = \frac{v_0}{g} = \frac{12}{9.8} = 1.22 \text{ seconds}$$

(c) Insert the formula for t at the time of maximum height into the formula for height:

$$h_{max} = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right)$$

$$h_{max} = -\frac{1}{2}\frac{v_0^2}{g} + \frac{v_0^2}{g}$$

$$h_{max} = \frac{1}{2}\frac{v_0^2}{g} = \frac{12^2}{2 \times 9.8} = 7.35 \text{ meters}$$

(d) Set h equal to zero:

$$0 = -\frac{1}{2}gt^2 + v_0 t$$

Solve for t:

$$\frac{1}{2}gt^2 = v_0 t$$

One solution will be where $t = 0$. This means that the height of the ball is zero at the instant it is thrown. To find the other value for t, divide both sides by t:

$$\frac{1}{2}gt = v_0$$

Solve for t:

$$t = \frac{2v_0}{g} = 2.44 \text{ seconds}$$

Note that the time until the ball hits the ground is twice as long as the time for it to reach its highest point.

$$(e)\ \frac{d^2h}{dt^2} = -g$$

8.

sparkle detergent : $Q_1 = 200 - 20P_1 + 6P_2$

dazzle detergent : $Q_2 = 30 - 10P_2 + 4P_1$

$Z = (P_1 - 5)(200 - 20P_1 + 6P_2) + (P_2 - 2)(30 + 4P_1 - 10P_2)$

$$\begin{array}{rrrrrr} 200P_1 & -20P_1^2 & +6P_1P_2 & -1000 & -30P_2 & \\ +100P_1 & & +4P_1P_2 & -60 & +30P_2 & -10P_2^2 \\ -8P_1 & & & & +20P_2 & \end{array}$$

The profit function is:

$Z = 292P_1 - 20P_1^2 + 10P_1P_2 - 1060 + 20P_2 - 10P_2^2$

Find the partial derivatives:

$$Z_{P_1} = \frac{\partial Z}{\partial P_1} = 292 - 40P_1 + 10P_2 = 0$$

$$Z_{P_2} = \frac{\partial Z}{\partial P_2} = 10P_1 + 20 - 20P_2 = 0$$

Write the equation system as:

$-40P_1 + 10P_2 = -292$

$10P_1 - 20P_2 = -20$

Divide the second equation by two. The new equation system is:

$-40P_1 + 10P_2 = -292$

$5P_1 - 10P_2 = -10$

Add the two equations together:

$-35P_1 = -302$

$$P_1 = \frac{-302}{-35} = 8.629 \quad \text{round to } 8.63$$

Put this value into the second equation to find P_2:

$5 \times 8.629 - 10P_2 = -10$

$43.15 + 10 = 10P_2$

$$P_2 = \frac{53.15}{10} = 5.315 \quad \text{round to } 5.32$$

To double check, put the value for P_1 into the first equation:

$-40 \times 8.629 + 10P_2 = -292$

$10P_2 = 345.16 - 292 = 53.16$

$$P_2 = \frac{53.16}{10} = 5.316 = \quad \text{round to } 5.32$$