Bus 3700 Quantitative Analysis
Practice Exercise Answers
Equation Systems and Linear Programming

1. (a) equation 1: slope$= -45/72 = -0.625$
y intercept$=540/72 = 7.500$; x intercept$=540/45 = 12.000$
equation 2: slope$= -20/58 = -0.345$
y intercept$=642/58 = 11.069$; x intercept$=642/20 = 32.100$

(c) matrix notation:

$$\begin{pmatrix} 45 & 72 \\ 20 & 58 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 540 \\ 642 \end{pmatrix}$$

determinant$=45 \times 58 - 20 \times 72 = 1170$

(d) To eliminate x, multiply first equation by 4 and multiply second equation by 9:

$$180x + 288y = 2160$$

$$180x + 522y = 5778$$

Subtract second equation from first equation:

$$-234y = -3618$$

$$y = -3618/(-234) = 15.462$$

Insert this value for y into the first equation:

$$x = (540 - 72 \times 15.462)/45 = -12.738$$

2. (a) To eliminate x, multiply first equation by 65 and multiply second equation by 34:

$$2210x + 5200y = 117000$$

$$2210x + 1020y = 71400$$

Subtract second equation from first equation:

$$4180y = 45600$$

$$y = 45600/4180 = 10.909$$

Insert this value for y into the first equation:

$$x = (1800 - 80 \times 10.909)/34 = 27.273$$

(b) In order for there to be an infinite number of solutions, the determinant must be zero; this will happen if:

$$34k - 80 \times 65 = 0$$

$$34k = 5200$$

$$k = 152.941$$

There will be an infinite number of solutions if the y intercepts are the same for each equation. The y intercept of the first equation is 1800/80, and the y intercept of the second equation is g/k, where the value of k must be 152.941 (see above). Then, this equation must be true if there are to be an infinite number of solutions:

$$\frac{1800}{80} = \frac{g}{152.941}$$

Solve the equation for g:

$$g = \frac{1800 \times 152.941}{80} = 3441.17$$

In this equation system:

$$34x + 80y = 1800$$

$$65x + 152.941y = 3441.17$$

any solution of one of the equations is also a solution for the other equation.

(c) Again, the determinant must be zero, so k must be 152.941. The value of g can be anything other than the value given in part (b).

3.(a)

$$\begin{pmatrix} 5 & 0 & 8 \\ 0 & 6 & 8 \\ 5 & 6 & 0 \end{pmatrix} \begin{pmatrix} 40 \\ 50 \\ 60 \end{pmatrix} = \begin{pmatrix} (5 \times 40 + 0 \times 50 + 8 \times 60) \\ (0 \times 40 + 6 \times 50 + 8 \times 60) \\ (5 \times 40 + 6 \times 50 + 0 \times 60) \end{pmatrix}$$

$$= \begin{pmatrix} 680 \\ 780 \\ 500 \end{pmatrix}$$

(b) $$\begin{pmatrix} 7 & 3 \\ 2 & 8 \end{pmatrix} \begin{pmatrix} 5 & 9 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} (7 \times 5 + 3 \times 6) & (7 \times 9 + 3 \times 4) \\ (2 \times 5 + 8 \times 6) & (2 \times 9 + 8 \times 4) \end{pmatrix}$$

$$= \begin{pmatrix} 53 & 75 \\ 58 & 50 \end{pmatrix}$$

$$\text{(c)} \quad \begin{pmatrix} 9.8 & 5.5 & 9.4 \\ 1.3 & 6.0 & 12.3 \\ 5.6 & 4.5 & 1.7 \end{pmatrix} \begin{pmatrix} 0.6 & 3.5 & 4.9 \\ 9.4 & 1.2 & 9.6 \end{pmatrix}$$

can't multiply a (3 by 3) matrix by a (2 by 3) matrix

$$\text{(d)} \quad \begin{pmatrix} 2 & 1 & 0 \\ 8 & 9 & 10 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1.6 & -0.4 & 1 \\ -2.2 & 0.8 & -2 \\ 0.7 & -0.3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (2.0 \times 1.6 + 1.0 \times -2.2 + 0.0 \times 0.7) & (2.0 \times -0.4 + 1.0 \times 0.8 + 0.0 \times -0.3) & (2.0 \times 1.0 + 1.0 \times -2.0 + 0.0 \\ (8.0 \times 1.6 + 9.0 \times -2.2 + 10.0 \times 0.7) & (8.0 \times -0.4 + 9.0 \times 0.8 + 10.0 \times -0.3) & (8.0 \times 1.0 + 9.0 \times -2.0 + 10.0 \\ (1.0 \times 1.6 + 2.0 \times -2.2 + 4.0 \times 0.7) & (1.0 \times -0.4 + 2.0 \times 0.8 + 4.0 \times -0.3) & (1.0 \times 1.0 + 2.0 \times -2.0 + 4.0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4. The two matrices from (d) happen to be inverses of each other (a lucky break for us; otherwise we would have needed to use Excel to find the inverse). Therefore, the solution to the equation system comes from this matrix multiplication:

$$\begin{pmatrix} 2 & 1 & 0 \\ 8 & 9 & 10 \\ 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 50 \\ 310 \\ 64 \end{pmatrix}$$

$$= \begin{pmatrix} (2 \times 50 + 1 \times 310 + 0 \times 64) \\ (8 \times 50 + 9 \times 310 + 10 \times 64) \\ (1 \times 50 + 2 \times 310 + 4 \times 64) \end{pmatrix}$$

$$= \begin{pmatrix} 410 \\ 3830 \\ 926 \end{pmatrix}$$

5. optimum: $x = 300$, $y = 80$. (The graphs for problems 5, 6, 7, and 8 are on the internet.)

6. optimum: $x = 12$, $y = 14$.

absolue value of slope=$p_x/p_y = p_x/5$:

price range for x	|isocost slope|	optimal solution
> 10	> 2	(0,38)
10 to 5.625	2 to 1.125	(12,14)
5.625 to 2.5	1.125 to .5	(20,5)
< 2.5	$< .5$	(30,0)

7. (a) maximize $1.10x + 1.30y$, subject to:

$$12x + 16y \leq 768$$
$$x + y \leq 60$$
$$x \leq 50$$
$$y \leq 40$$

(b)

$$12x + 16y + s_1 = 768$$
$$x + y + s_2 = 60$$
$$x + s_3 = 50$$
$$y + s_4 = 40$$

(c) (0,0); (0,40); (10.67,40); (48,12); (50,10); (50,0)

(d) Optimal solution: (48,12); value: \$68.4. Constraints 1 and 2 are binding at the optimum, so s_1 and s_2 are zero.

$$x + s_3 = 50$$

Since $x = 48$, $s_3 = 2$.

$$y + s_4 = 40$$

Since $y = 12$, $s_4 = 28$. At the optimum, four variables are nonzero: $x = 48$, $y = 12$, $s_3 = 2$, $s_4 = 28$. This is the same as the number of constraints.

8. (a)

$$x + y + s_1 = 460$$
$$12x + 20y + s_2 = 8,040$$
$$8x + 3y + s_3 = 3,000$$

(b) optimum: $x = 145$, $y = 315$.

(c) absolue value of slope=$c_x/c_y = 10/c_y$:

profit range for y	|isocost slope|	optimal solution
> 16.6667	< 0.6	(0,402)
16.6667 to 10	0.6 to 1	(145,315)
10 to 3.75	1 to 2.6667	(324,136)
< 3.75	> 2.6667	(375,0)

(d) At the optimum, constraints 1 and 2 are binding. Constraint 3 is non-binding, so its shadow price is zero.

To find the shadow price for constraint 1, increase the capacity of that constraint by 1 and solve this equation system:

$$\begin{pmatrix} 12 & 20 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8040 \\ 461 \end{pmatrix}$$

Result: $x = 147.500, y = 313.500$, value = 5,864.00

The shadow price is the difference between the new value and the original value: $5,864.00 - 5,860.00 = 4.00$.

To find the shadow price for constraint 2, increase the capacity of that constraint by 1 and solve this equation system:

$$\begin{pmatrix} 12 & 20 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8041 \\ 460 \end{pmatrix}$$

Result: $x = 144.875, y = 315.125$, value = 5,860.50

The shadow price is the difference between the new value and the original value: $5,860.50 - 5,860.00 = 0.50$.