Business 3700 Quantitative Analysis
Practice Examination I Answers

1. $$P = 820 - 12Q$$

$$TR = PQ = 820Q - 12Q^2$$

$$\frac{dTR}{dQ} = MR = 820 - 24Q$$

$$0 = 820 - 24Q$$

$$24Q = 820$$

$$Q = \frac{820}{24}$$

$$Q = 34.1667$$

$$P = 410$$

2. Set marginal cost equal to price:

$$\frac{dTC}{dQ} = MC = 15Q^2 - 18Q + 12 = 444$$

$$15Q^2 - 18Q - 432 = 0$$

Use the quadratic formula:

$$Q = \frac{18 + \sqrt{(-18) \times (-18) - [4 \times (15) \times (-432)]}}{30}$$

$$Q = \frac{18 + \sqrt{324 - (-25,920.0)}}{30.0}$$

$$Q = \frac{18 + \sqrt{26,244.0}}{30.0}$$

$$Q = \frac{18 + 162}{30.0}$$

$$Q = \frac{180}{30}$$

$$Q = 6$$

3. $Z =$

$$\begin{array}{rrrrrr} 1,105P_x & -5{P_x}^2 & +3P_xP_y & & & \\ +900P_x & & & -198,900 & +1,290P_y & \\ & & +4P_xP_y & & -540P_y & -6{P_y}^2 \\ -1,080P_x & & & -348,300 & +1,620P_y & \end{array}$$

$$Z = 925P_x - 5{P_x}^2 + 7P_xP_y + 2,370P_y - 6{P_y}^2 - 547,200$$

Pretend that P_y is a constant, and find the partial derivative with respect to P_x:

$$Z_{P_x} = \frac{\partial Z}{\partial P_x} = 925 - 10P_x + 7P_y$$

Pretend that P_x is a constant, and find the partial derivative with respect to P_y:

$$Z_{P_y} = \frac{\partial Z}{\partial P_y} = 2,370 - 12P_y + 7P_x$$

(Note: the curly d's in $\partial Z/\partial P_y$ are another notation that can be used for partial derivatives.)

Set both partial derivatives to zero:

$$-10P_x + 7P_y = -925$$

$$7P_x - 12P_y = -2,370$$

To eliminate x, multiply the top equation by 7 and multiply the bottom equation by 10 :

$$-70P_x + 49P_y = -6,475$$

$$70P_x - 120P_y = -23,700$$

Add the two equations:

$$-71P_y = -30,175$$

$$P_y = \frac{-30,175}{-71} = 425$$

$$P_x = 390$$

4. $$TR = P_1Q_1 + P_2Q_2$$

Substitute in the expressions for P_1 and P_2:

$$TR = (1,400 - 2Q_1)Q_1 + (2,400 - 8Q_2)Q_2$$

$$TR = 1,400Q_1 - 2{Q_1}^2 + 2,400Q_2 - 8{Q_2}^2$$

Substitute $480 - Q_1$ in place of Q_2:

$$TR = 1,400Q_1 - 2{Q_1}^2 + 2,400(480 - Q_1) - 8(480 - Q_1)^2$$

$$TR = 1,400Q_1 - 2{Q_1}^2 + 2,400(480 - Q_1) - 8(480^2 - 2 \times 480Q_1 + {Q_1}^2)$$

$$TR = 1,400Q_1 - 2{Q_1}^2 + 2,400(480 - Q_1) - 8(230,400 - 960Q_1 + {Q_1}^2)$$

$$TR = 1,400Q_1 - 2{Q_1}^2 + 1,152,000 - 2,400Q_1 - 1,843,200 + 7,680Q_1 - 8{Q_1}^2$$

Take the derivative of TR with respect to Q_1:

$$\frac{dTR}{dQ_1} = 1,400 - 4Q_1 - 2,400 + 7,680 - 16Q_1$$

$$\frac{dTR}{dQ_1} = 6,680 - 20Q_1$$

Set the derivative equal to 0:

$$0 = 6,680 - 20Q_1$$

$$6,680 = 20Q_1$$

$$Q_1 = 334$$

$$Q_2 = 146$$

$$P_1 = 732$$

$$P_2 = 1,232$$

Note: at the optimal point, the marginal revenues are:

$$1,400 - 2 \times 2 \times 334 = 64$$

and:

$$2,400 - 2 \times 8 \times 146 = 64$$

The two marginal revenues are equal at the optimal point. If the marginal revenues were not equal, then you could increase revenue by producing more of the produce with the higher marginal revenue, and less of the product with the lower marginal revenue.

5. Solve the second equation for y:

$$C = ax^2 + bxy$$

$$C - ax^2 = bxy$$

$$\frac{C - ax^2}{bx} = y$$

Insert the formula for y into the formula for V:

$$V = x^2 y$$

$$V = x^2 \left(\frac{C - ax^2}{bx} \right)$$

Cancel an x:

$$V = x \left(\frac{C - ax^2}{b} \right)$$

Multiply the x across:

$$V = \frac{Cx - ax^3}{b} = \frac{C}{b}x - \frac{a}{b}x^3$$

Find the derivative dV/dx:

$$\frac{dV}{dx} = \frac{C}{b} - \frac{3a}{b}x^2$$

Set the derivative equal to zero:

$$0 = \frac{C}{b} - \frac{3a}{b}x^2$$

Multiply both sides b:

$$0 = C - 3ax^2$$

Solve for x:

$$3ax^2 = C$$

$$x^2 = \frac{C}{3a}$$

$$x = \sqrt{\frac{C}{3a}}$$